MODAL BOUNDARY CONDITIONS FOR WAVEGUIDES OF ARBITRARY CROSS-SECTION WITH SCN TLM

J. L. Herring, M. Righi and W. J. R. Hoefer

NSERC/MPR Teltech Research Chair in RF Engineering, Department of Electrical & Computer Engineering, University of Victoria, P.O. Box 3055, Victoria, B.C., Canada, V8W 3P6.

ABSTRACT

Modal absorbing boundary conditions are applied to a graded mesh using the hybrid node, and to homogeneous waveguides of arbitrary cross-section. The best performance is obtained with rectangular waveguides but examples show that the method can also be used for other geometries. A T-junction circular - to - sidecoupled rectangular waveguide example is given.

INTRODUCTION

The transmission-line matrix (TLM) method of electromagnetic analysis with the symmetrical condensed node (SCN) [1] is well established. When modelling discontinuities in waveguides, there is a need to keep the TLM mesh as small as possible by terminating the input and output ports with absorbing boundary conditions (ABC's). Wideband ABC's are essential if full advantage is to be taken of the ability of time domain methods to obtain frequency characteristics over a very wide frequency spectrum from a single simulation. An ideal ABC can be implemented by convolving each of the voltage pulses, incident from link-lines intersecting the ABC, with the impulse response of an infinite length of waveguide. However, in the case of homogeneous waveguides, where there are well defined modes with a frequency independent configuration, the computational effort can be reduced by first extracting the mode amplitude, performing a single convolution, and then re-injecting the mode shape with the resulting amplitude. The characteristic impulse response of the TLM network can be considered as a discrete time domain modal Green's function. The method was first implemented for the dominant mode in a rectangular waveguide by Eswarappa *et al* [2,3] and was extended to the multi-mode case by Righi *et al* [4,5]. The approach has also been used in FDTD [6]. In addition to ABC's, the same technique can be used to represent efficiently waveguide stubs [7]. In this paper, modal ABC's are first applied to a graded mesh using the hybrid node formulation. Secondly, the approach is extended to homogeneous waveguides of arbitrary cross-section where the mode configuration may not be known analytically.

APPLICATION TO RECTANGULAR WAVEGUIDES WITH HYBRID NODE GRADED MESH

For rectangular waveguides, the modal field distribution is well known. In the application of modal boundary conditions to TLM, it is convenient to work directly with the voltage pulses, rather than with the total tangential field components. This is possible because the link-line voltages are a linear combination of the electric and magnetic fields. For a graded mesh using the hybrid node, three different impedances are assigned to the three sets of link-lines contributing to each component of the magnetic field [8]. The modal Green's function is determined by the timestep and the node dimension perpendicular to the boundary; it is independent of the mesh grading in the cross-section.

Typical Green's functions for the dominant mode in a WR28 waveguide are shown in the frequency domain in fig. 1, for the cases of (*i*) cubic nodes with the maximum timestep, (*ii*) cubic nodes with half the maximum timestep, and (*iii*) cuboid nodes of twice the length in the perpendicular direction and with the maximum timestep. As the frequency increases, the curves tend towards the link-line reflection coefficient needed to match the TLM mesh to free-space:

$$\rho_l = \frac{\Delta l_{min} - \Delta l}{\Delta l_{min} + \Delta l} \qquad \text{where} \qquad \frac{l_{imn}}{t} = 2c\ \Delta t$$

Δl is the perpendicular node spacing, Δt is the timestep and c is the speed of light.

For accurate extraction of the mode amplitudes, the node dimensions must be of constant size in each quarter-sine (90˚) variation of the field profile, in both directions. If this condition is not met, there can be numerical coupling between modes. The restriction can be relaxed if it is known that only a single mode is incident upon the boundary.

APPLICATION TO HOMOGENEOUS WAVEGUIDES OF ARBITRARY CROSS-SECTION

For homogeneous waveguides of arbitrary cross-section, the mode configuration can be obtained from two previous two-dimensional simulations. It is convenient to use a slice of three-dimensional nodes for this purpose. The cutoff frequencies are obtained from the first simulation and then, in the second, the Fourier transform of the appropriate voltage pulses is computed at these frequencies over the whole plane. If this involves sampling at the excitation point, care must be taken to ensure that the excitation does not introduce any unwanted artifacts into the result, for example, by applying a suitable window function in time. Even when the mode configuration is known analytically, it can still be appropriate to obtain the coupling matrix from a TLM simulation, so that the field profile exactly matches the discrete geometry of the system.

To form the modal coupling matrix, the complex output from the Fourier transform must be converted to real numbers. The magnitude can be taken if the field is orientated in a single direction, otherwise the complex numbers can be resolved at an arbitrary phase angle, provided it is away from the field minimum. Often it is appropriate to simply take the real part. The resulting matrix can be used directly to convert from mode amplitudes to TLM voltage pulses. To convert back to mode amplitudes, a new matrix is formed in which each element is multiplied by the area of the corresponding node, and then a scaling factor is applied so that the conversions *to modes* and *from modes* are consistent. The voltage pulses incident on the boundary are then multiplied by the elements of this matrix.

For modes of type $TE_{n,0}$ in rectangular waveguides aligned with the coordinate axes, voltage pulses of one polarization are considered. For modes of type $TE_{n,m}$ (m>0), there is a corresponding $TM_{n,m}$ mode. When working directly with the TLM voltage pulses, both TE and TM modes are processed together by using separate modal Green's functions for the vertical→vertical, horizontal→horizontal and vertical ↔horizontal coupling. This involves four convolutions since the vertical→horizontal and horizontal→vertical convolutions are independent. The same approach can also be used for other waveguide cross-sections. However, for some geometries, both vertical and horizontal voltage pulses are needed to describe only a single mode. For the $TE_{1,1}$ mode in a circular waveguide, it can be convenient to obtain the mode amplitude from just the vertical voltage pulses (with the mode aligned top-to-bottom) since the horizontal field is not large. In this case, only vertical→vertical and vertical→horizontal convolutions are needed.

The reflection coefficient of practical ABC's can be calculated as an S-parameter, from a knowledge of the total voltage and the incident voltage at the input port, as shown in fig. 2a. The reference structure is used to obtain the incident voltage, and the length l_1 is chosen so that voltage pulses reflected from the ends of the waveguide are not visible at the output point, within the duration of the simulation. If the reflection coefficient is worse than about -50dB, it can be calculated from the VSWR, as shown in fig. 2b. To do this, the Fourier transform is calculated along a line of nodes in the centre of the waveguide, and the magnitude is scanned for the minimum and maximum values. The length l_2 is chosen so that there is at least one cycle at the lowest frequency of interest. For very small reflections, the VSWR is close to unity and the method becomes inaccurate.

Typical results are shown in fig. 3 for a ridged waveguide modelled at two different resolutions (32 by 16 nodes and 64 by 32 nodes) and for a circular waveguide. The geometry of the ridged waveguide is shown in fig. 4. The performance of the ABC improves as the resolution is increased. Also, if the width w is reduced, the results get worse. This suggests that it is the poor description of the external corners in the TLM mesh that is the problem. The circular waveguide was modelled on a Cartesian grid with a *staircase* approximation and the same performance was obtained with mesh sizes of 20 by 20 and 40 by 40 nodes. Here, there is a large field concentration away from the edges, and the effect of the external corners in the staircase is not as pronounced.

For rectangular waveguides, the performance of modal ABC's approaches the limit of numerical precision. Spurious reflections as low as -120dB are obtained with single precision arithmetic and -300dB with double precision. In the absence of external corners, an impulsive excitation with the spatial distribution of any valid mode will always maintain exactly the same shape as it propagates. When external corners are present, they tend to drag the field, distorting the mode shape. It is felt that a corner correction that can eliminate this phenomenon will allow much better performance to be obtained with this type of ABC. Nevertheless, modal ABC's even without the benefit of any corner correction give performance comparable with other types of ABC.

EXAMPLE

The ABC's for uniform waveguides of arbitrary shape have been applied to the circular ports of the circular-to-rectangular T-junction shown in fig. 5 [9]. A standard modal ABC for rectangular waveguides was used on port 3. Only the dominant modes were considered. The ABC's were therefore placed away from the junction, to ensure that all higher order modes would be sufficiently decayed. The entire structure was discretized on a cubic mesh with 28 cells in the diameter of the circular waveguide. The S-parameters are shown in fig. 6, along with results obtained with a mode matching approach. Similar TLM results can be obtained with one-way equation ABC's [10], although there is always the potential for instabilities with this type of boundary.

CONCLUSIONS

The proposed boundary condition provides an alternative method for terminating homogeneous waveguides of arbitrary shape. The procedure does not require any optimization of parameters and it is stable. It can be used with multiple modes, whether propagating or evanescent, if the ABC's need to be placed close to a discontinuity. In addition, the modal approach can be extended to the characterization of one-ports or n-port components.

REFERENCES

1. P. B. Johns, "A symmetrical condensed node for the TLM method", *IEEE Trans. MTT*, vol. 35, no. 4, pp 370–377, Apr 1987.
2. Eswarappa, G. I. Costache and W. J. R. Hoefer, "Transmission line matrix modeling of dispersive wide-band absorbing boundary conditions with time-domain diakoptics for S-parameter extraction", *IEEE Trans. MTT*, vol. 38, no. 4, pp 379–386, Apr 1990.
3. C. Eswarappa and W. J. R. Hoefer, "Diakoptics and wideband dispersive absorbing boundaries in the 3-D TLM method with symmetrical condensed node", *IEICE Trans.*, vol. E74, pp 1242–1250, May 1991.
4. M. Righi, M. Mongiardo, R. Sorrentino and W. J. R. Hoefer, "Efficient TLM diakoptics for separable structures", *1993 IEEE MTT-S Int. Symp. Dig.*, pp 425–428.
5. M. Righi and W. J. R. Hoefer, "Efficient 3D-SCN-TLM diakoptics for waveguide components", *IEEE Trans. MTT*, vol. 42, no. 12, pp 2381–2385, Dec 1994.
6. F. Moglie, T. Rozzi, P. Marcozzi and A. Schiavoni, "A new termination condition for the application of FDTD techniques to discontinuity problems in close homogeneous waveguides", *IEEE Microwave and Guided Wave Letters*, vol. 2, no. 12, pp 475–477, Dec 1992.
7. M. Righi and W. J. R. Hoefer, "Efficient hybrid TLM/mode matching analysis of packaged components", *1996 IEEE MTT-S Int. Symp. Dig.*, pp 447–450.
8. R. Scaramuzza and A. J. Lowery, "Hybrid symmetrical condensed node for the TLM method", *Electronics Letters*, vol. 26, no. 23, pp 1947–1948, Nov 1990.
9. P. Krauss and F. Arndt, "Rigorous mode-matching method for the modal analysis of the T-junction circular to sidecoupled rectangular waveguide", *1995 IEEE MTT-S Int. Symp. Dig.*, pp 1355–1358.
10. C. Eswarappa and W. J. R. Hoefer, "One-way equation absorbing boundary condition for 3-D TLM analysis of planar quasi-planar structures", *IEEE Trans. MTT*, vol. 42, no. 9, pp 1669–1677, Sept 1994.

ACKNOWLEDGEMENTS

The authors wish to thank C. Eswarappa for useful discussions. This research has been funded by the Natural Sciences and Engineering Research Council of Canada, the Science Council of British Columbia, MPR Teltech Inc. of Burnaby, B.C., and the University of Victoria. Dr. Righi has been supported by an NSERC Post-doctoral fellowship.

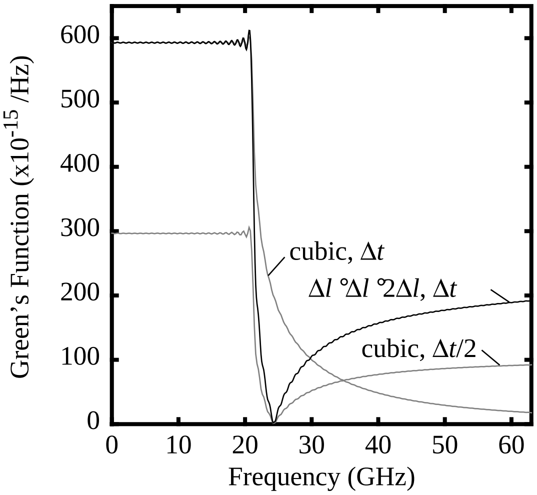

Fig. 1 – Modal Green's functions for hybrid node

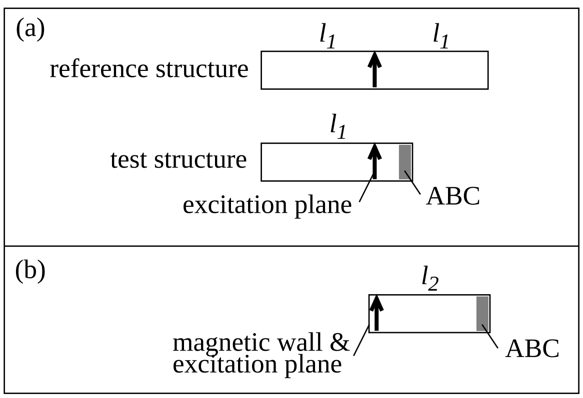

Fig. 2 – Calculation of reflection coefficient: (a) with reference structure, (b) from VSWR

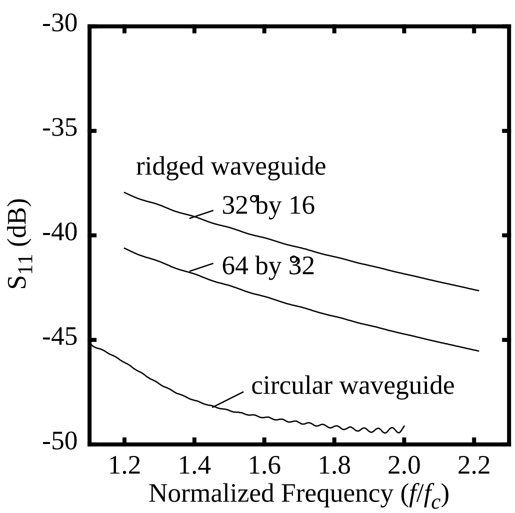

Fig. 3 – Spurious reflection coefficient for ridged and circular waveguides terminated with modal ABC's

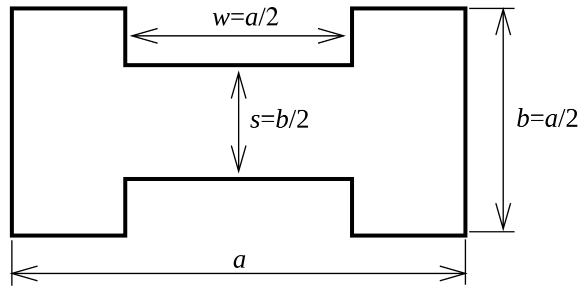

Fig. 4 – Geometry of ridged waveguide

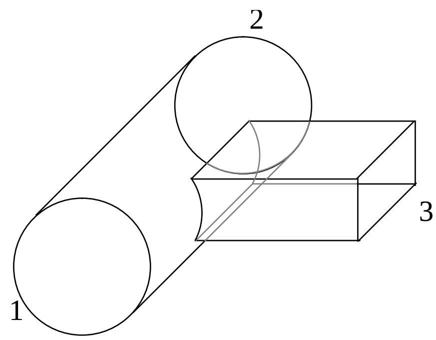

Fig. 5 – Geometry of the circular to sidecoupled rectangular waveguide T-junction. Radius: 7.0 mm, rectangular waveguide: WR-62 (16.0 mm by 8.0 mm)

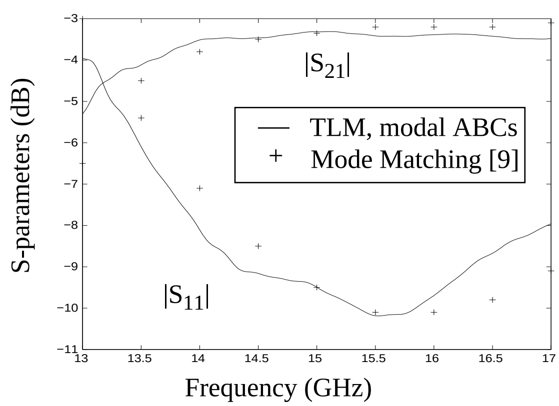

Fig. 6 – Magnitude of the S-paramters of the circular to sidecoupled rectangular waveguide T-junction.

0-7803-4603-6/97/$5.00 (c) IEEE